ECE 388

Automatic Control

LAB 1

State Variables

Objectives: Many engineering systems have many inputs and many outputs, as shown in Figure 1. The state variable method is a very important tool in analyzing systems and understanding such highly complex systems. Thus, the state variable model is more general than the single-input, single-output model, such as a transfer function. We will learn how MATLAB helps in solving these type of problems.

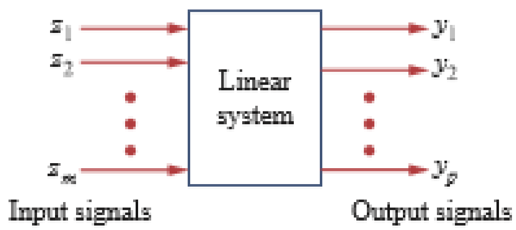


Figure 1

List of Equipment/Software

MATLAB

Representation of State Equations

The standard way to represent the state equations is to arrange them as a set of first-order differential equations:

$$\dot{\boldsymbol{x}} = \boldsymbol{A}\boldsymbol{x} + \boldsymbol{B}\boldsymbol{z}$$

$\boldsymbol{x}(\dot{\boldsymbol{t}}) = \begin{bmatrix} x_1(t) \\ x_2(t) \\ \cdot \\ \cdot \\ \cdot \\ x_n(t) \end{bmatrix}$, state vector representing n state variables where $\dot{\boldsymbol{x}}$ represents the first derivative

$\boldsymbol{z}(\boldsymbol{t}) = \begin{bmatrix} z_1(t) \\ z_2(t) \\ \cdot \\ \cdot \\ \cdot \\ z_m(t) \end{bmatrix}$, input vector representing m input

A and B are respectively nxn and nxm matrices. In addition to the state equation, we need the output equation. The complete state model or state space is

$$\dot{\boldsymbol{x}} = \boldsymbol{A}\boldsymbol{x} + \boldsymbol{B}\boldsymbol{z}$$

$$y = Cx + Dz$$

Where,

$$y(t) = \begin{bmatrix} y_1(t) \\ y_2(t) \\ \cdot \\ \cdot \\ \cdot \\ y_p(t) \end{bmatrix}$$, output vector representing p input

C and D are, respectively,pxn and pxm matrices. For the special case of single-input single-output, n=m=p=1.

Steps to Apply the State Variable Method to Circuit Analysis

1. Select the inductor current i and capacitor voltage v as the state variables, making sure they are consistent with the passive sign convention.

2. Apply KCL and KVL to the circuit and obtain circuit variables (voltages and currents) in terms of the state variables. This should lead to a set of first-order differential equations necessary and sufficient to determine all state variables.

3. Obtain the output equation and put the final result in state-space representation.

Lab Exercise

Step1:

Consider the circuit in Figure 2, Determine the state-Space representation of the circuit.

Where v_s is the input and i_x is the output. Take $R = 1\Omega, C = 0.25\ F, and\ L = 0.5\ H$.

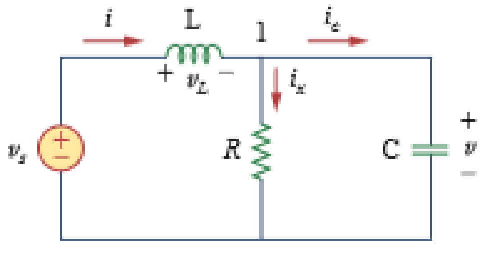


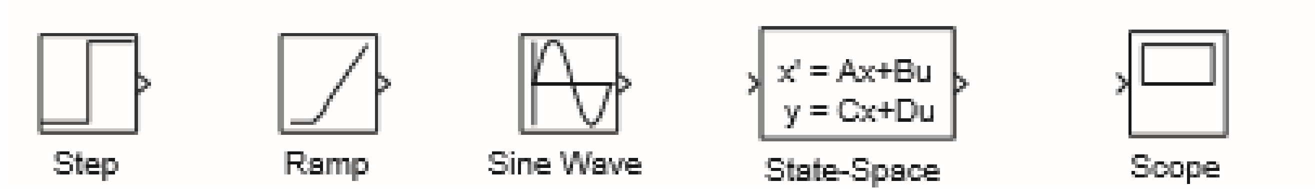
Figure 2

Step2:

We want to use Simulink for simulating the state space model found in step1.

a) Use **step input signal** and find simulation results
b) Use **ramp input signals** and find simulation results
c) Use **sinusoidal input signal** and find simulation results.

HINT: Use the following blocks in the Simulink Library Browser



Step3: Now consider the following State Space Model and apply step 2 (a,b and c) for this model.

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} -2 & -2 \\ 2 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 1 \\ -1 \end{bmatrix} u$$

$$y = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

$$\dot{x} = \begin{bmatrix} -3 & 1 & -1 \\ 2 & -2 & 1 \\ -3 & 4 & -5 \end{bmatrix} x + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} u$$

$$y = \begin{bmatrix} 2 & -5 & 0 \end{bmatrix} x$$

$$\dot{x} = 5\,x + 13\,u$$

$$y = x - 3\,u$$